AFRL

An Optimal Observation Policy in Opportunistic Navigation

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Facts and Implications

- GNSS Vulnerabilities
  - Multipath, partial or complete obscuration
  - Dynamics, platforms, and environments
  - Jamming and non-deliberate interferences
- Navigation with Signals of Opportunity
  - Abundant and free to use
  - Available with geometry and frequency diversities
  - More powerful than GPS

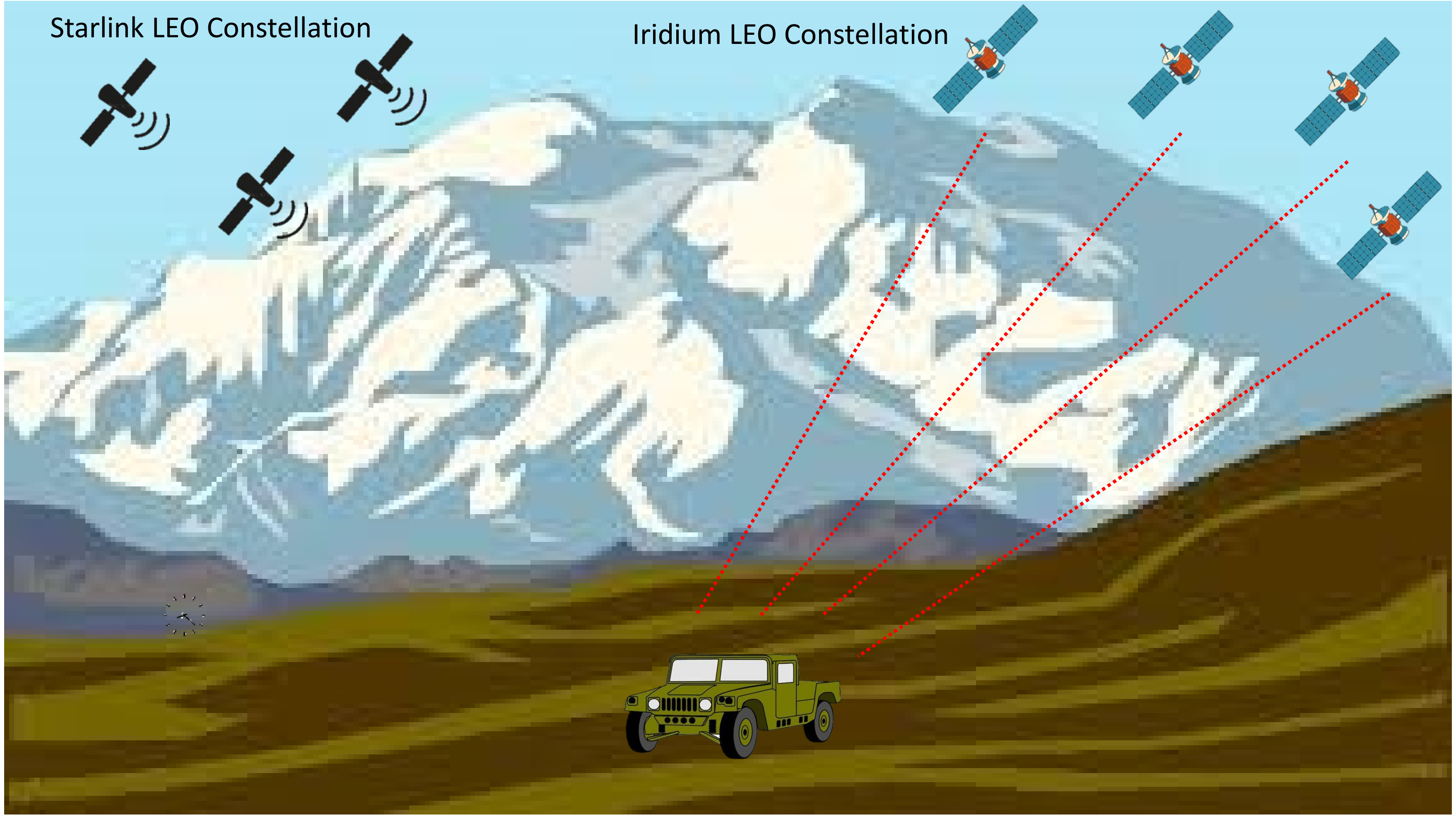
GNSS – Global Navigation Satellite Systems
GPS – Global Positioning System

Premises and Boundary Conditions

- Focus on Multisensory Hybridization
  - Ad-Hoc vs. No Infrastructures
  - Platform Dynamics
  - Sensor Integration
- Focus on LEO-based PNT
  - Opportunistic navigation with LEO satellite signals
  - Cost of observations; e.g., Doppler and Doppler rates
  - Provision observation policy given prediction accuracy level
  - Tradeoff measurement strategies and on-line computations

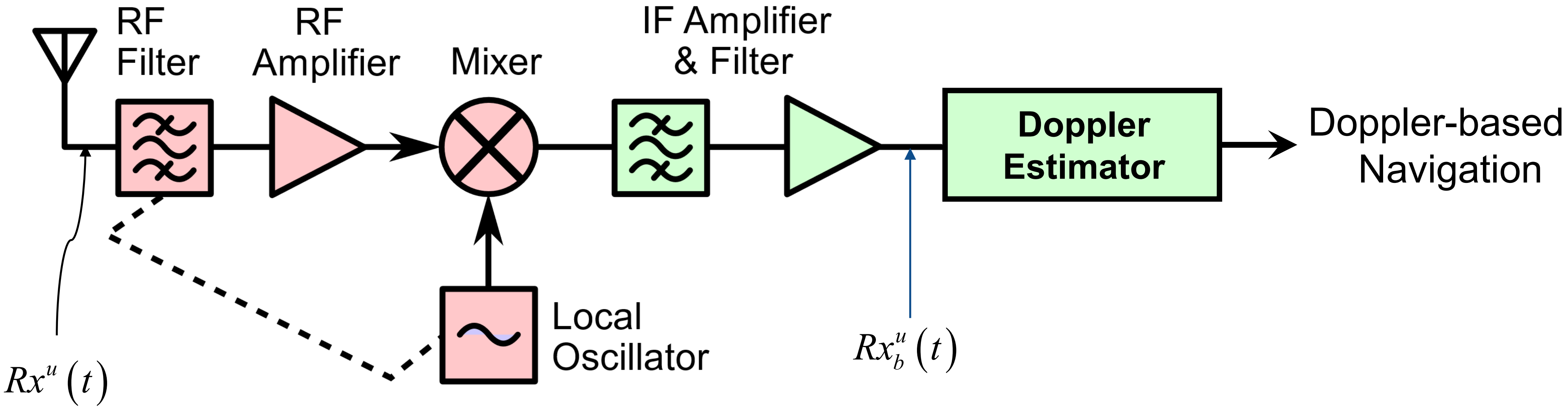
PNT – Positioning, Navigation, and Timing
LEO – Low Earth Orbit

Navigation with LEO Satellite Signals of Opportunity



Pass Prediction – Satellites Visible at Elevation Angles Between 8° and 90°

Multi-Constellation Receiver Architecture

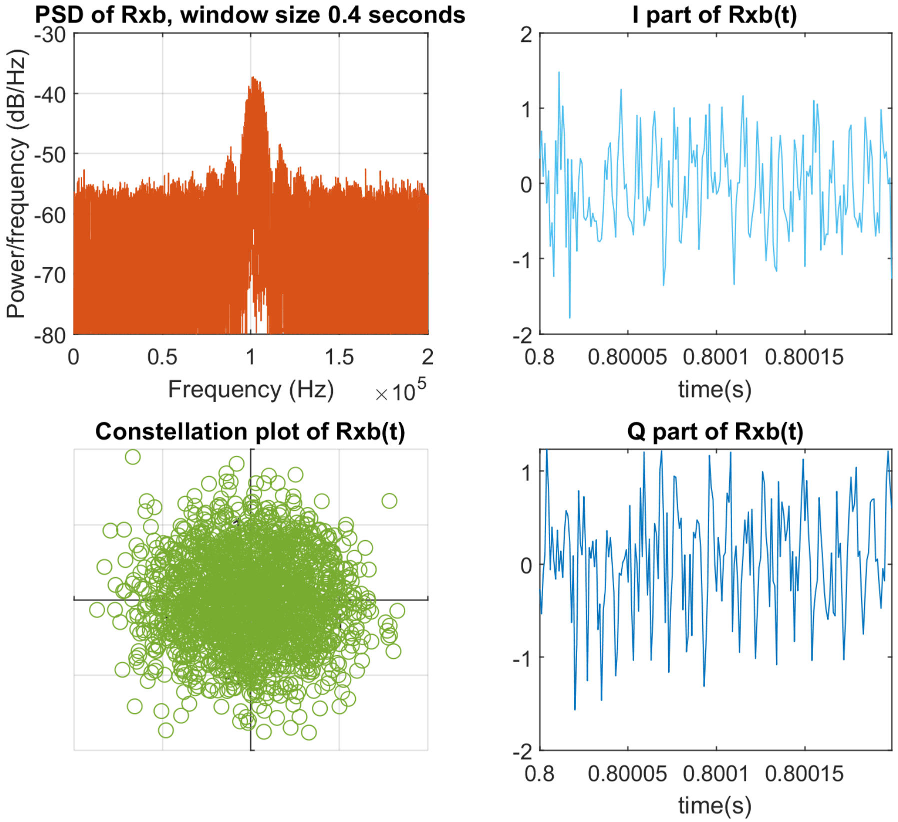


$$Rx^u(t) = \sqrt{P_{r,u}} \cos\left\{ \sum_{l_u=1}^{L_u} \left[2\pi \left(f_{c,u} + f_{l_u}\right) \left(t - \frac{R_{l_u}(t)}{c} \right) + d_{l_u} \left(t - \frac{R_{l_u}(t)}{c} \right) \frac{2\pi}{M_{l_u}} \right] \right\}$$

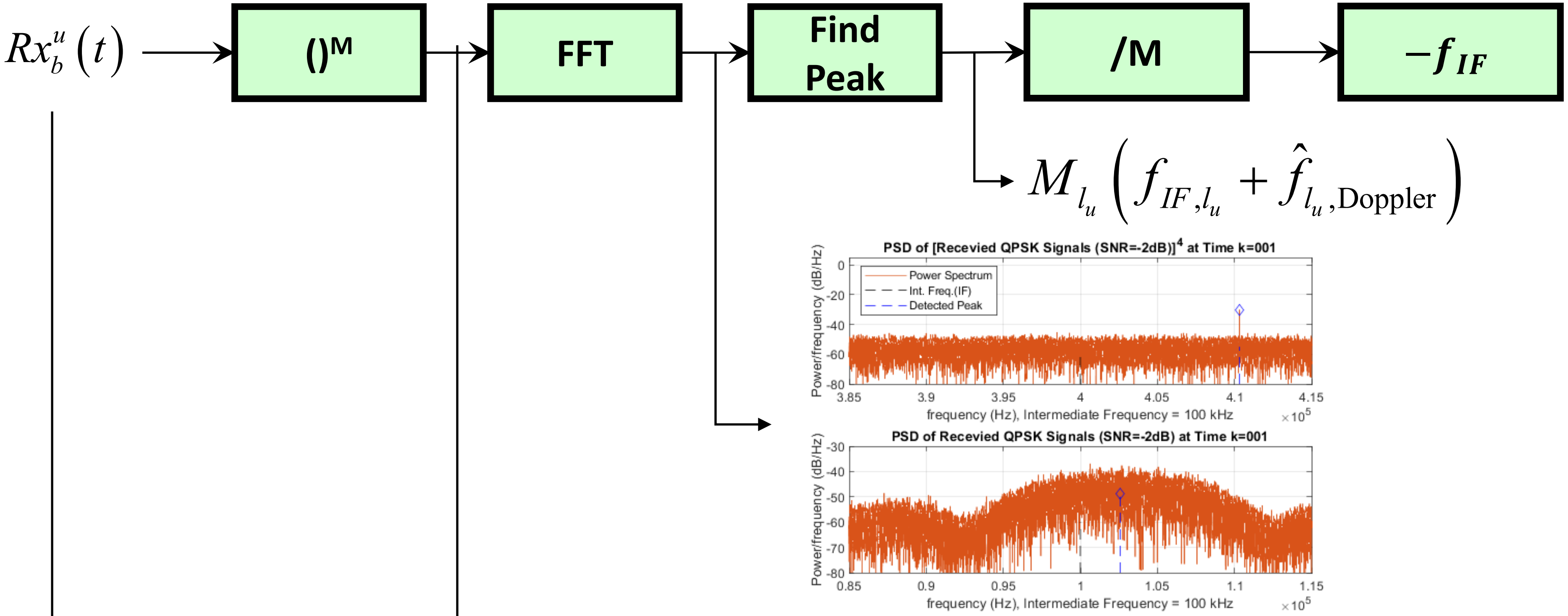
u – uth - LEO Constellation
L_u – Number of Visible Satellites
c – Speed of Light
R_{lu} – Distance Between Satellite & Rx
M_{lu} – M-ary Shift Keying Modulation Index

Received Baseband Signals

$$Rx_b^u(t) = \sqrt{P_{g,u} P_{r,u}} \exp\left\{ j \sum_{l_u=1}^{L_u} \left[2\pi f_{IF,l_u} t - 2\pi f_{c,u} \frac{R_{l_u}(t)}{c} + d_{l_u}\left(t - \frac{R_{l_u}(t)}{c}\right) \frac{2\pi}{M_{l_u}} \right] \right\}$$



Estimation of Doppler and Doppler Rate



$$M_{l_u}\left(f_{IF,l_u}+\hat{f}_{l_u,\text{Doppler}}\right)$$

$$P_{r,u}^2 P_{g,u}^2 \exp\left\{ j\left[2M_{l_u}\pi f_{IF,l_u} t - 2M_{l_u}\pi f_{c,u}\frac{R_{l_u}(t)}{c} + d_{l_u}\left(t-\frac{R_{l_u}(t)}{c}\right)2\pi \right]\right\}$$

$$Rx_b^u(t)=\sqrt{P_{g,u}P_{r,u}}\exp\left\{ j\sum_{l_u=1}^{L_u}\left[2\pi f_{IF,l_u} t - 2\pi f_{c,u}\frac{R_{l_u}(t)}{c} + d_{l_u}\left(t-\frac{R_{l_u}(t)}{c}\right)\frac{2\pi}{M_{l_u}} \right]\right\}$$

Pseudorange Rate Measurements

$$z_{l_u}(k) = c\frac{\hat{f}_{l_u,\text{Doppler}}(k)}{f_{c,u} + f_{IF,l_u}}; \qquad l_u = 1,\ldots,L_u$$

- Re-indexed and Concatenating Notation

$$z_j(k) = \left\{z_{l_u}(k)\right\}_{l_u=1}^{L_u}; \qquad j \triangleq u = 1,\ldots,\mathrm{U}$$

- Measurement Model

$$z_j(k) \simeq \frac{\dot{r}_j^T(k)\left[r_{Rx}(k) - r_j(k)\right]}{\left\|r_{Rx}(k) - r_j(k)\right\|} + c\Delta\dot{\delta}t_j + v_j(k); \qquad j = 1,\ldots,\mathrm{U}$$

r_{Rx} – Receiver Position
r_j – 3-D Position of Visible Satellites
$\dot{r}_j$ – 3-D Velocity of Visible Satellites
$\Delta\dot{\delta}t_j$ – Differential Rx & Satellite Clock Drifts

Simplified View of Sensor Structure

- Sensor Constraints

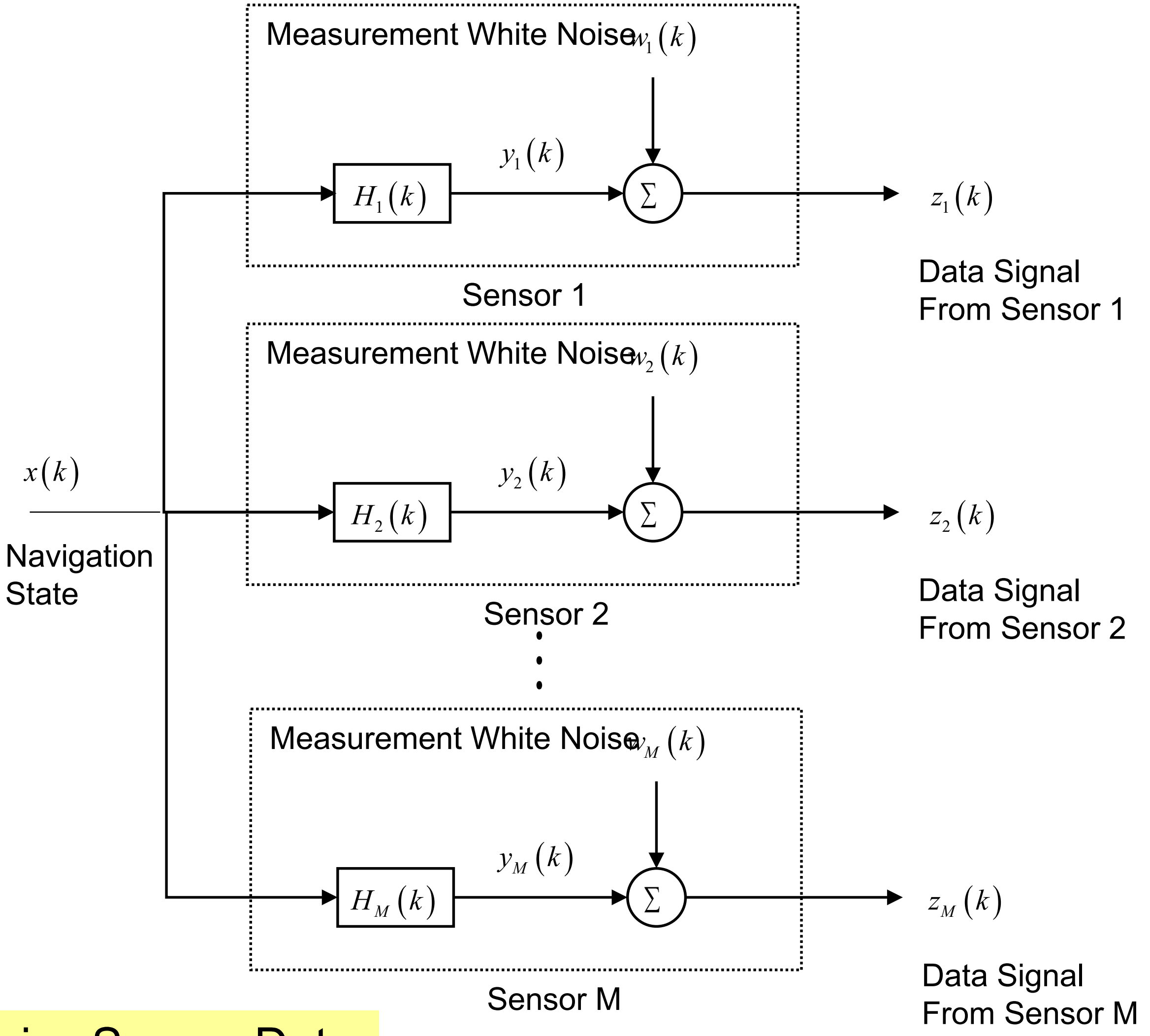
$$z_j(k) = y_j(k) + w_j(k) = H_j(k)x(k) + w_j(k)$$

where

$$y_j(k) = H_j(k)x(k); \qquad j = 1, \ldots, M$$

- Measurement White Noises

$$E\{w_j(k)\} = 0$$
$$E\{w_j(k)w_j^T(l)\} = W_j(k)\delta(k,l)$$



Partial Observable and Noisy Sensor Data

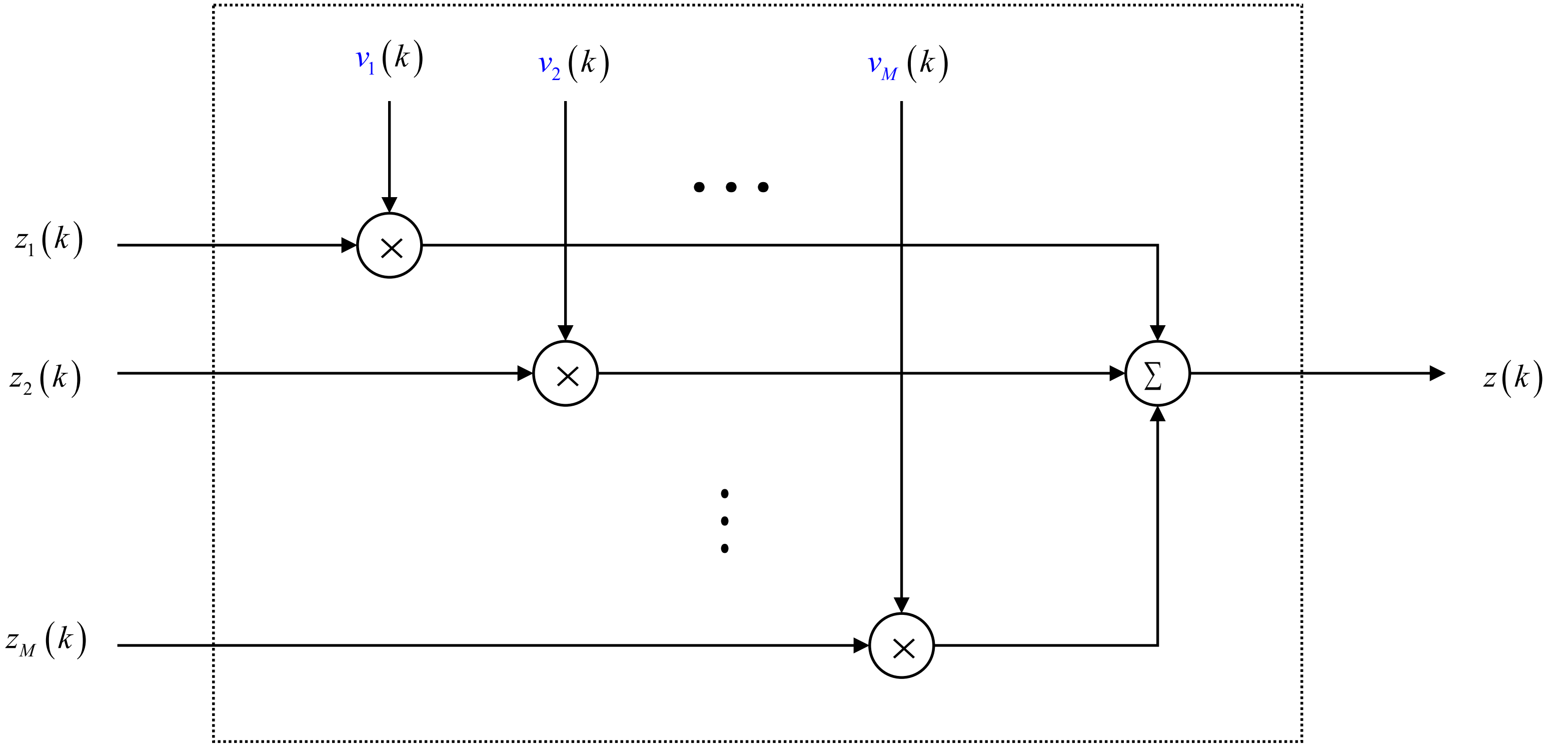
Sensor Selection

- Sensor Selection

$$v(k) = \begin{bmatrix} v_1(k) \\ v_2(k) \\ \vdots \\ v_M(k) \end{bmatrix}$$

where

$$\begin{cases} \sum_{i=1}^{M} v_i(k) = 1 \\ 0 \le v_i(k) \le 1 \end{cases}$$



- Selected Sensor Observations

$$z(k) = v_1(k) z_1(k) + v_2(k) z_2(k) + \cdots + v_M(k) z_M(k)$$

Resources or Instruments Required to Process Sensor Observations

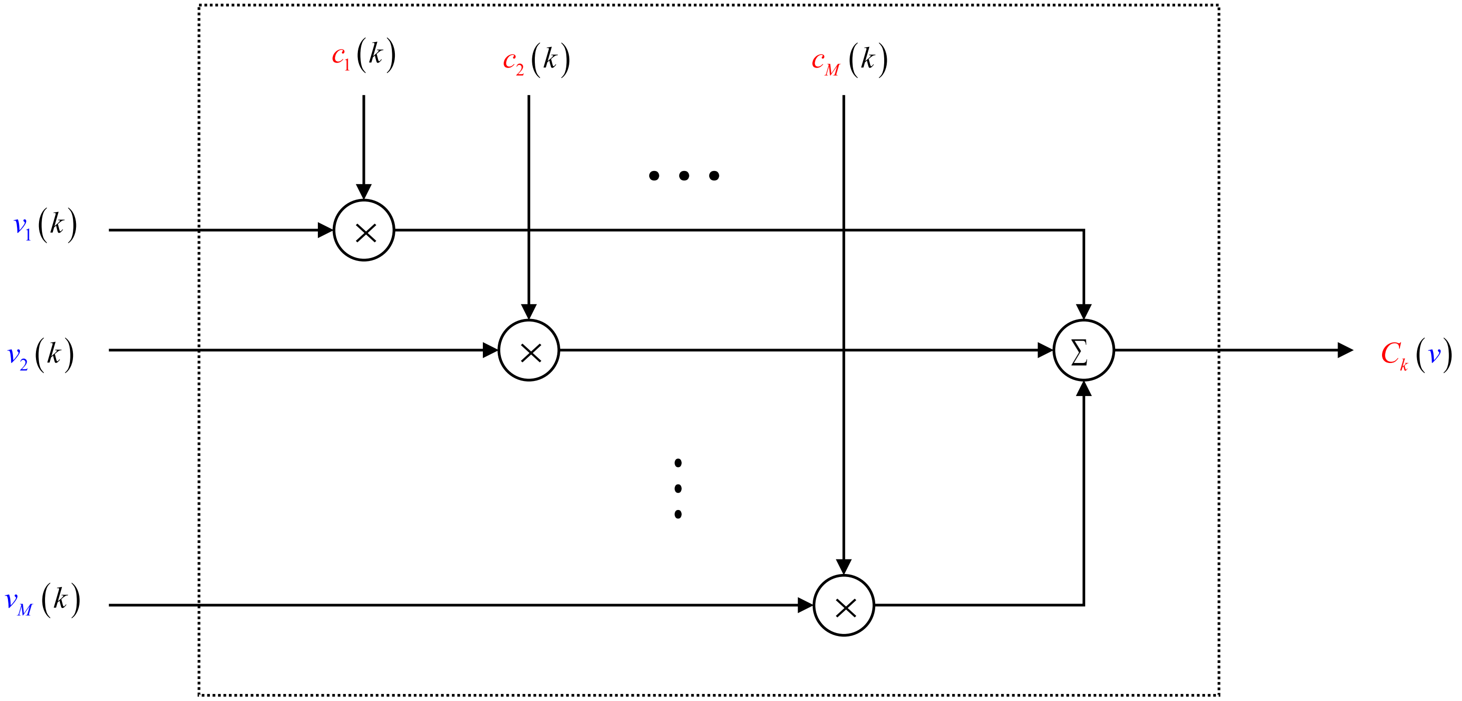
Cost of Observations

- Per-Unit-of-Time Cost

$$C_k(v) = \sum_{j=1}^{M} c_j(k) v_j(k)$$

where

$$0 \le c_j(k); \qquad j = 1, \ldots, M$$



- Total Cost of Observation Policy

$$C(v) = \sum_{k=0}^{N-1} C_k(v)$$

Inherent Costs Associated to Sensor Observation Strategies

Prediction Requirements

- State Space Model for Receiver Positions / Satellite & Receiver Clock Drifts

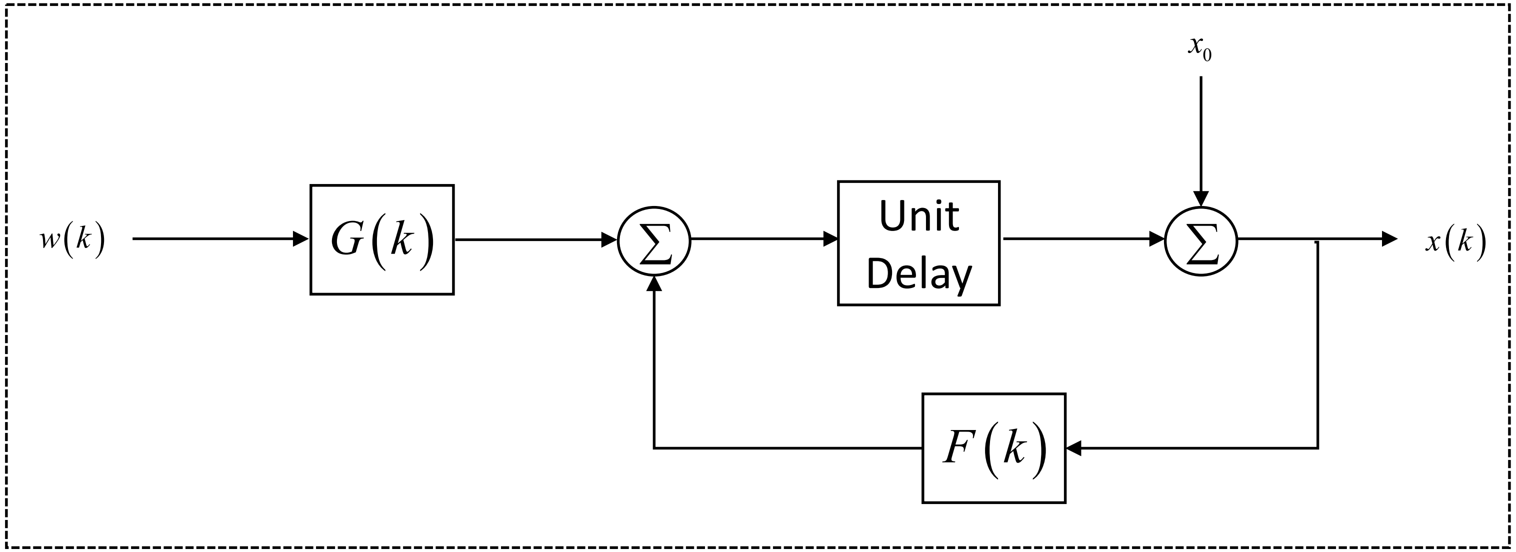
$$x(k+1) = F(k)x(k) + G(k)w(k); \qquad x(0) = x_0$$

where $\{w(k)\}$ is white in the strict sense

$$E\{w(k)\} = 0$$
$$\text{cov}(w(k), w(l)) = W(k)\delta(k,l)$$

- Common Gauss-Markov Navigation Process

$$x(k) = \Phi(k,0)x_0 + \sum_{l=0}^{k-1} \Phi(k,l+1)G(l)w(l)$$



Prediction Requirements

- Diverse Measurements Carried Out

$$[0, N]$$

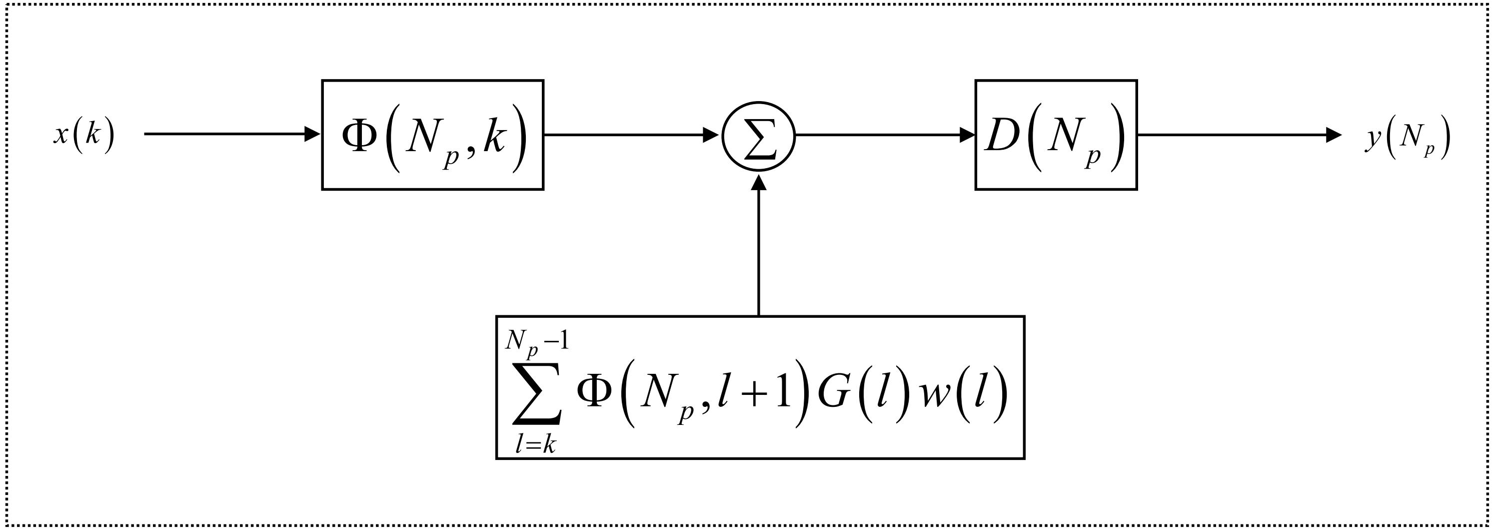
- Predicting Important States; e.g., Receiver Positions, LEO satellite & Receiver Clock Drifts

$$y(k) = D(k)x(k)$$

at $k = N_p,\ N_p \geq N$

- Prediction Interval

$$N_p - N \geq 0$$



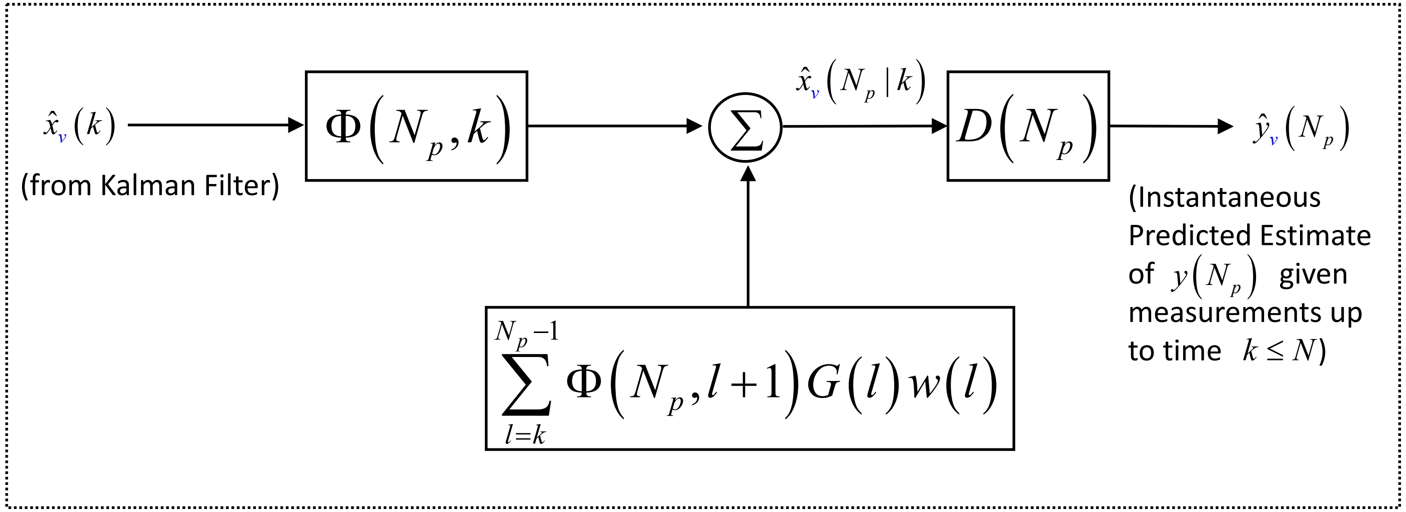
Prediction Requirements

- Denote Estimates of $y(N_p)$ by $\hat{y}(N_p)$
- Accuracy of Prediction Depending on

$$E\left\{y(N_p)-\hat{y}(N_p)\right\}\cong 0$$

$$\hat{J}(N_p)=E\left\{\left[y(N_p)-\hat{y}(N_p)\right]^T\left[y(N_p)-\hat{y}(N_p)\right]\right\}$$

- Structure of Predictor



The Smaller $\hat{J}(N_p)$ The More Accurate The Prediction

Statement of Optimization Problem

- Given the Integrated Receiver / Satellite & Receiver Clock Drift System

$$x(k+1) = F(k)x(k) + G(k)w(k); \qquad x(0) = x_0$$

- Focusing on Premise Variables

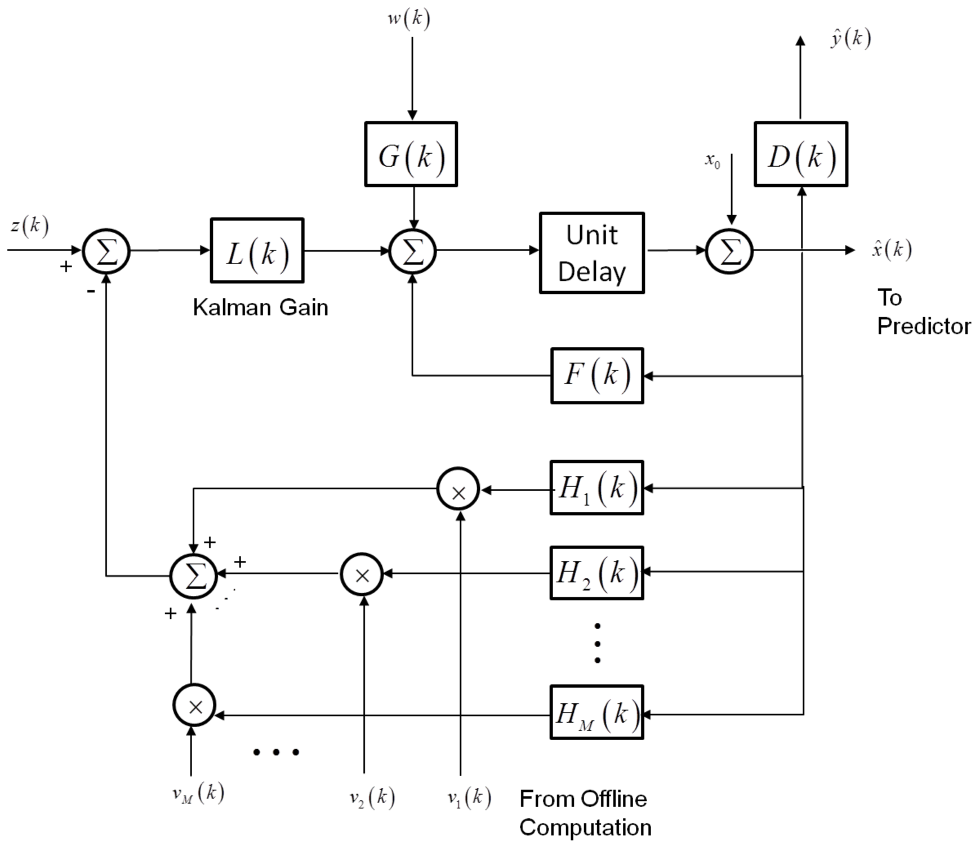
$$y(k) = D(k)x(k)$$

- Leveraging Doppler Measurements

$$z_j(k) = H_j(k)x(k) + w_j(k); \qquad j = 1, \ldots, M$$

- Optimizing Observation Policy

$$v_1(k), v_2(k), \ldots, v_M(k); \qquad k \in [0, N]$$



Statement of Optimization Problem

- Optimal Observation Policy Subject to

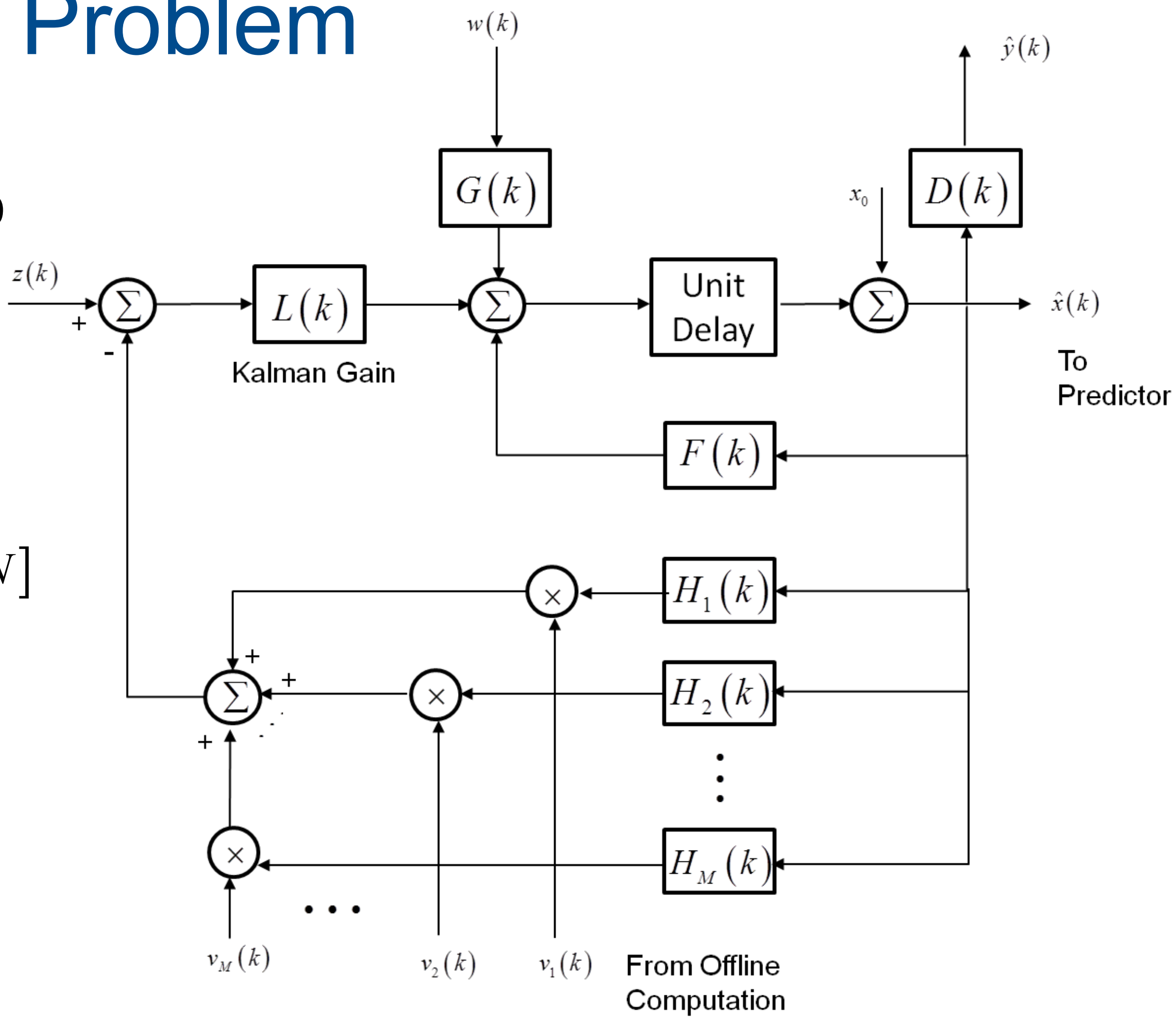
$$\begin{cases} \sum_{i=1}^{M} v_i(k) = 1 \\ 0 \le v_i(k) \le 1 \end{cases}$$

which leads to $E\{y(N_p) - \hat{y}(N_p)\} \cong 0 \text{ for } k \in [0, N]$

and the multi-objective utility

$$J \triangleq \alpha C(v) + (1-\alpha)\hat{J}(N_p); \qquad 0 \le \alpha \le 1$$

is minimized.



Tradeoffs Between Total Observation Cost and Mean-Square Error

State Estimation

- Let $v(k),\ k \in [0, N]$ denote a fixed observation policy during time interval $[0, N]$
- It follows the least mean-square error estimate of the common process

$$\hat{x}_v^+(k) \equiv \hat{x}_v(k \mid k) \triangleq E\{x(k) \mid Z_k\}$$

where $Z_k = \{z(0), z(1), \ldots, z(k)\}$

- Similarly, one-step predictor is given by

$$\hat{x}_v^-(k) \equiv \hat{x}_v(k \mid k-1) \triangleq E\{x(k) \mid Z_{k-1}\}$$

- Respective estimation error covariance become

$$P_v^+(k) \equiv P_v(k \mid k) \triangleq E\left\{\left[x(k) - \hat{x}_v^+(k)\right]\left[x(k) - \hat{x}_v^+(k)\right]^T \mid Z_k\right\}$$

$$P_v^-(k) \equiv P_v(k \mid k-1) \triangleq E\left\{\left[x(k) - \hat{x}_v^-(k)\right]\left[x(k) - \hat{x}_v^-(k)\right]^T \mid Z_{k-1}\right\}$$

State Estimation

$$\hat{x}_{\nu}^{+}(k)=\hat{x}_{\nu}^{-}(k)+L_{\nu}(k)\left[z(k)-\sum_{i=1}^{M}\nu_i(k)H_i(k)\hat{x}_{\nu}^{-}(k)\right]$$

$$P_{\nu}^{+}(k)=P_{\nu}^{-}(k)-P_{\nu}^{-}(k)\sum_{i=1}^{M}\nu_i(k)H_i^T(k)\left[\sum_{j=1}^{M}\nu_j(k)H_j(k)P_{\nu}^{-}(k)\sum_{l=1}^{M}\nu_l(k)H_l^T(k)+\sum_{m=1}^{M}\nu_m^2(k)W_m(k)\right]^{-1}\sum_{i=1}^{M}\nu_i(k)H_i(k)P_{\nu}^{-}(k)$$

- Time-Update Step:

$$\hat{x}_{\nu}^{-}(k+1)\equiv\hat{x}_{\nu}(k+1\,|\,k)\triangleq E\left\{x(k+1)\,|\,Z_k\right\}=F(k)\hat{x}_{\nu}^{+}(k)$$

$$P_{\nu}^{-}(k+1)\equiv P_{\nu}(k+1\,|\,k)\triangleq E\left\{\left[x(k+1)-\hat{x}_{\nu}^{-}(k+1)\right]\left[x(k+1)-\hat{x}_{\nu}^{-}(k+1)\right]^T\right\}=F(k)P_{\nu}^{+}(k)F^T(k)+G(k)W(k)G^T(k)$$

- Kalman Gain

$$L_{\nu}(k)=P_{\nu}^{-}(k)\sum_{i=1}^{M}\nu_i(k)H_i^T(k)\left[\sum_{j=1}^{M}\nu_j(k)H_j(k)P_{\nu}^{-}(k)\sum_{l=1}^{M}\nu_l(k)H_l^T(k)+\sum_{m=1}^{M}\nu_m^2(k)W_m(k)\right]^{-1}$$

State Prediction

$$\hat{x}_v^+(N_p) = \Phi(N_p, N)\hat{x}_v^+(N) + \sum_{l=N}^{N_p-1} \Phi(N_p, l+1) G(l) w(l)$$

where

$$\Phi(k,l) = \begin{cases} F(k-1)F(k-2)\cdots F(l); & k > l \geq 0 \\ I; & k = l \end{cases}$$

- Important Parameter Prediction

$$\hat{y}_v(N_p) = D(N_p)\hat{x}_v^+(k)$$

- It leads to

$$E\{y(N_p) - \hat{y}_v(N_p)\} = 0$$

- Covariance of Prediction Error

$$S_v(N_p) \triangleq E\left\{\left[y(N_p) - \hat{y}_v(N_p)\right]\left[y(N_p) - \hat{y}_v(N_p)\right]^T\right\} = D(N_p)P_v^+(N_p)D^T(N_p)$$

$$P_v^+(N_p) = \Phi(N_p, N)P_v^+(N)\Phi^T(N_p, N)$$

Reformulation of the Optimization Problem

$$P_v^+(k) = P_v^-(k) - P_v^-(k)\sum_{i=1}^{M} v_i(k) H_i^T(k)\left[\sum_{j=1}^{M} v_j(k) H_j(k) P_v^-(k) \sum_{l=1}^{M} v_l(k) H_l^T(k) + \sum_{m=1}^{M} v_m^2(k) W_m(k)\right]^{-1} \sum_{i=1}^{M} v_i(k) H_i(k) P_v^-(k)$$

subject to

$$\begin{cases} \sum_{i=1}^{M} v_i(k) = 1; \quad \text{for all } k \in [0, N] \\ 0 \le v_i(k) \le 1; \quad \text{for all } k \in [0, N] \end{cases}$$

- Find the optimal observation policy

$$v_j^*(k)$$

such that the cost functional, with N fixed,

$$J = \alpha C(v) + (1-\alpha) Tr\left\{D(N_p)\Phi(N_p, N) P_v^+(N) \Phi^T(N_p, N) D^T(N_p)\right\}; \qquad 0 \le \alpha \le 1$$

Summary

- Conducting work of paradigm shift in alternative PNT
  - Opportunistic navigation with LEO satellite signals
  - Cost of observations; e.g., Doppler and Doppler rates
  - Provision observation policy given prediction accuracy level
  - Tradeoff measurement strategies and on-line computations
- Future Work
  - Model-based Assisting Deep Q Learning
  - Single Satellite Geolocation

Questions?